



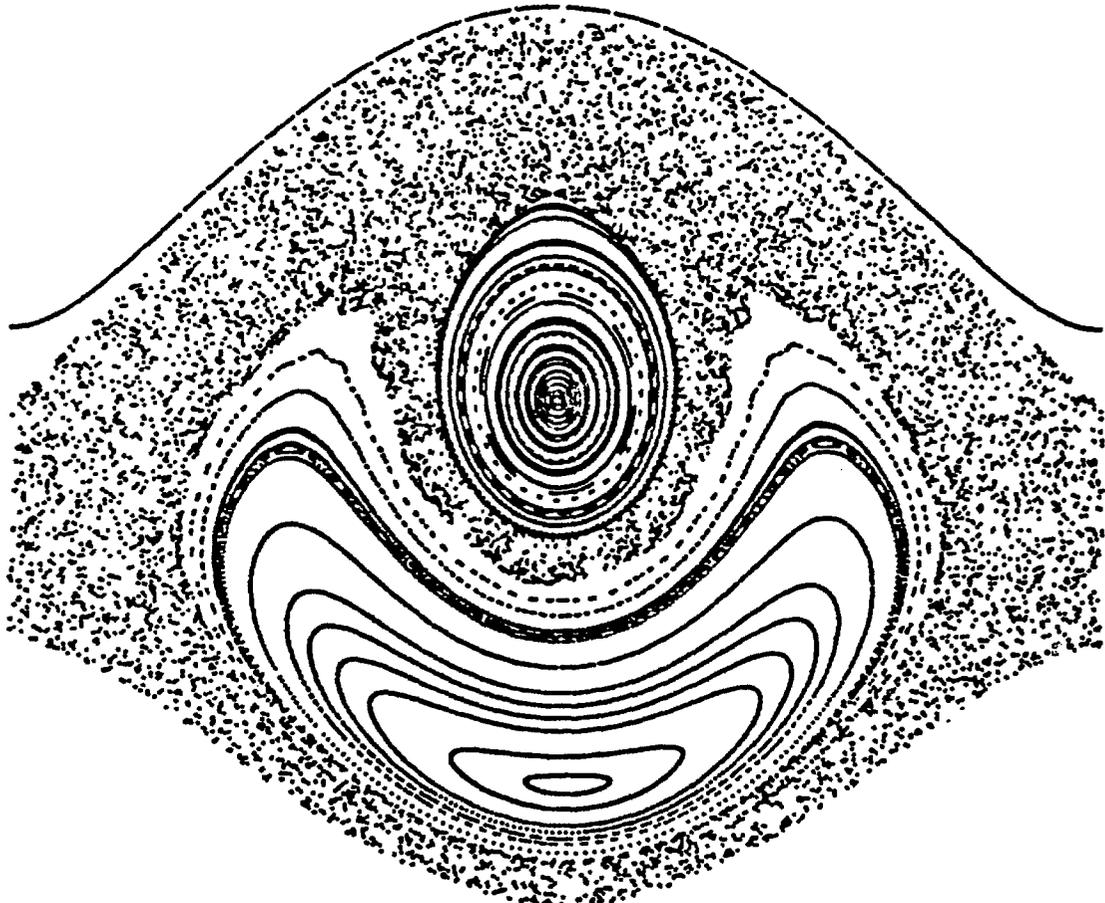
ACCELERATOR

PHYSICS

DEPARTMENT

TITLE: PROTON THERAPY ACCELERATOR (PTA) DESIGN CONSIDERATIONS

AUTHOR: Steve Peggs
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PROTON THERAPY ACCELERATOR (PTA) DESIGN CONSIDERATIONS

S. Peggs

ABSTRACT

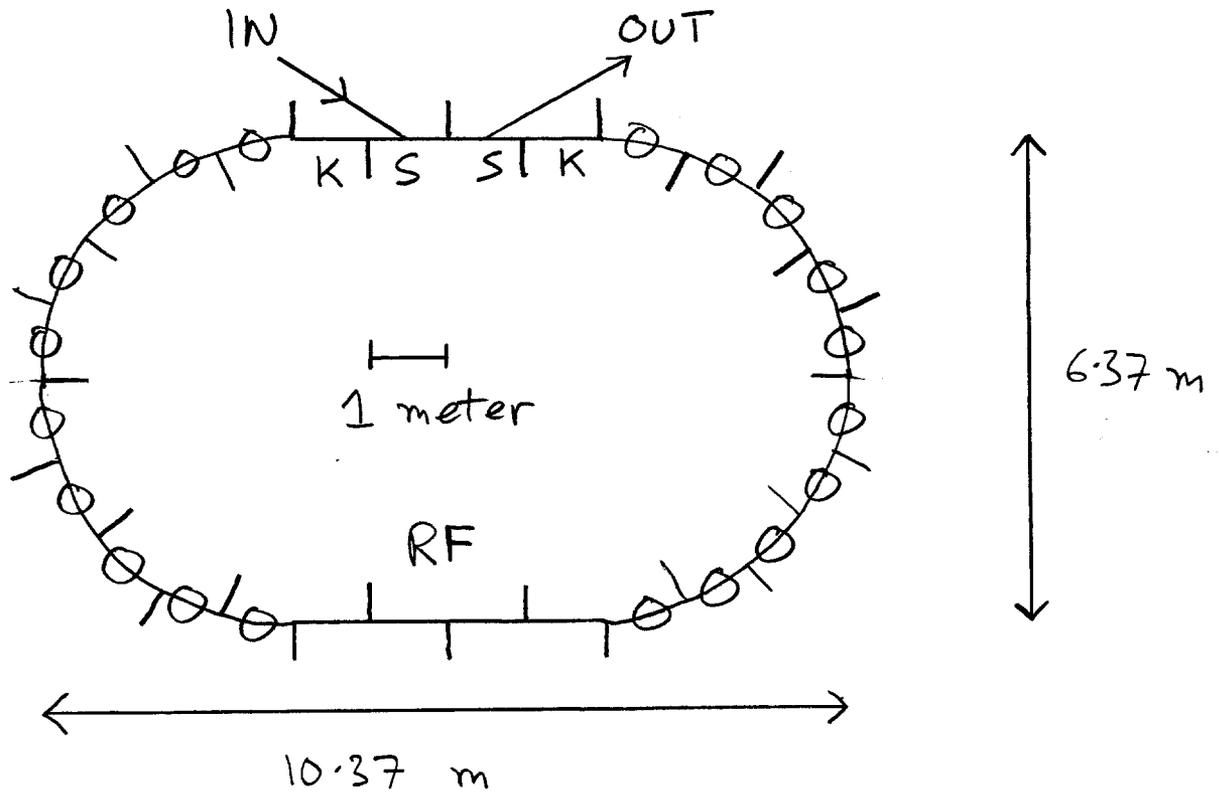
SEVERAL LINAC UPGRADE COMPONENTS - QUADRUPOLES, BPM'S, DIPOLE CORRECTORS - CAN BE USED FOR A PTA WITH MODEST REDESIGN IF THE 4 CM DIAMETER APERTURE IS ACCEPTABLE.

TYPICAL PARAMETERS FOR A RAPID CYCLING RESONANT SYNCHROTRON, MADE FROM A REGULAR SERIES OF FODO CELLS, ARE PRESENTED BASED ON THIS ASSUMPTION.

RAPID PROTOTYPING IS POSSIBLE IN THIS SCENARIO.

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SAMPLE MACHINE



Symbol	Quantity	Value
L (m)	half cell length	1.0
NA	$\frac{1}{2}$ cells per arc	10
$\phi_{1/2}$ (deg)	$\frac{1}{2}$ cell phase advance	36
β_{max} (m)	beta max	3.34
Δ_{max}^F (m)	disp. max (@ F quad)	2.35
Δ_{max}^D (m)	disp. max (@ D quad)	2.13
N_s	$\frac{1}{2}$ cells per straight	4
C (m)	circumference	28
B_0 (T)	dipole field	1.2
B_0' (T/m)	quad gradient	21.6
L_D (m)	dipole length	.628
L_Q (m)	quad length	.132
Q	tune	2.8
γ_T	transition gamma	≈ 2.0
ρ (m)	bend radius	2.0
R (m)	average arc radius	3.18
T_{ext} (GeV)	extraction energy	≤ 0.250
T_{inj} (GeV)	injection energy	0.015
θ_D (deg)	dipole bend angle	18

RANGE & DOSE

Assume $\frac{dE}{dx} = \frac{\rho z^2}{\beta^2} \left[\frac{dE}{dx} \right]_{\text{min}} = \frac{k \rho z^2}{\beta^2}$

where $\rho = \text{density} = 10^3 \text{ kg/m}^3$ for H_2O

$z = \text{ion charge} = 1$ for protons, below

$k = 1.4 \times 10^{-4} \text{ GeV}/(\text{kgm}/\text{m}^2)$ [empirical fit]

Now,

$$E = m + T = m \gamma$$

$$p = (E^2 - m^2)^{\frac{1}{2}} = m \beta \gamma$$

$$\beta = \frac{p}{E}$$

$$B\rho = \frac{p}{299792}$$

So, with $m = 0.938 \text{ GeV}/c^2$

T [GeV]	p [GeV/c]	β	$B\rho$ [Tsk-m]
.015	.168	.152	.562
.030	.239	.247	.798
.070	.369	.366	1.23
.250	.729	.614	2.43

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Measure x from stopping point, so

$$\frac{dx}{dE} = \frac{1}{k\rho z^2} \frac{E^2 - m^2}{E^2}$$

and integrating gives

$$\frac{T^2}{m+T} = k\rho z \cdot x$$

so that, solving the quadratic in T gives

$$T = \frac{(k\rho z)}{2} \left[x + \left(x^2 + \left(\frac{4m}{k\rho z} \right) x \right)^{\frac{1}{2}} \right]$$

and scaling with

$$x_c \equiv \frac{4m}{k\rho z} = 26.8 \text{ metres for protons in water}$$

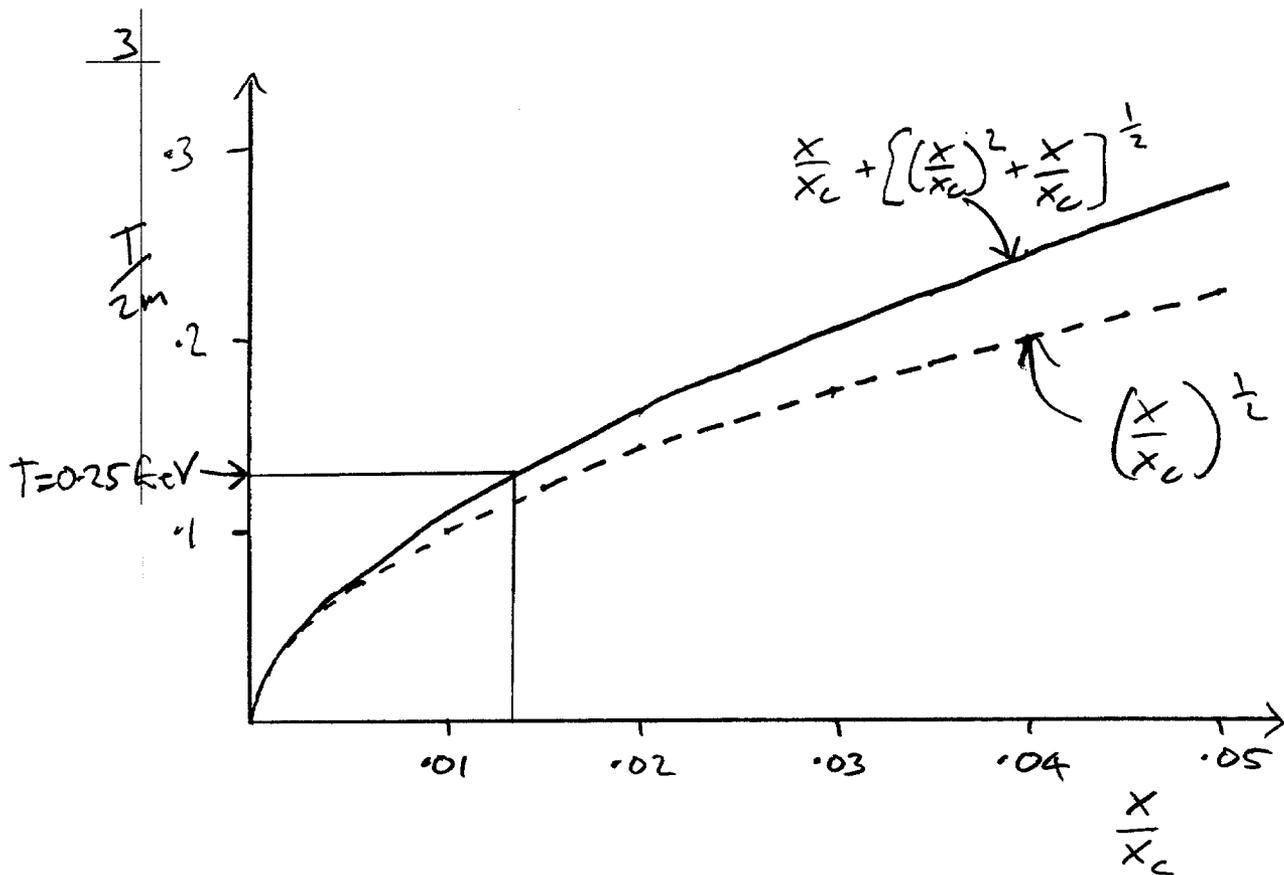
gives

$$T = 2m \left[\left(\frac{x}{x_c} \right) + \left\{ \left(\frac{x}{x_c} \right)^2 + \left(\frac{x}{x_c} \right) \right\}^{\frac{1}{2}} \right]$$

That is,

$$x \ll x_c, \quad T \approx 2m \left(\frac{x}{x_c} \right)^{\frac{1}{2}} = m^{\frac{1}{2}} (k\rho z)^{\frac{1}{2}} x^{\frac{1}{2}}$$

$$x \gg x_c, \quad T \approx 4m \left(\frac{x}{x_c} \right) = (k\rho z) x$$



So, it is a better than 10% approximation to use

$$T = 2m \left(\frac{x}{x_c} \right)^{1/2}$$

for $T < 0.250 \text{ GeV}$

For example, can say that R , the range, is

$$R = \left(\frac{T}{2m} \right)^2 x_c$$

$$[= 0.48 \text{ m @ } T = 0.25 \text{ GeV}]$$

instead of

$$R = \frac{\left(\frac{T}{2m} \right)^2 x_c}{1 + 2 \left(\frac{T}{2m} \right)}$$

$$[= 0.38 \text{ m @ } T = 0.25 \text{ GeV}]$$

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Using this approximation, the Bragg peak is illustrated by

$$\frac{dE}{dx} = \frac{dT}{dx} = \frac{m}{x_c^{1/2}} \frac{1}{x^{1/2}} = \frac{1}{2} \frac{T}{x}$$

So the variation in range with kinetic energy spread is

$$\boxed{\frac{\Delta R}{R} = 2 \frac{\Delta T}{T} \approx 4 \frac{\Delta p}{p}}$$

where the approximation

$$T = \frac{p^2}{2m} \text{ is valid } [= 0.283 \text{ keV when } p = 0.729 \frac{GeV}{c}$$

LINEAR RANGE SCANNING

Go to normalised coordinate y ,

$$\boxed{y = \frac{x}{x_c}}$$

$$T = 2m y^{1/2}$$

$$\boxed{\frac{dT}{dy} = \frac{m}{y^{1/2}}}$$

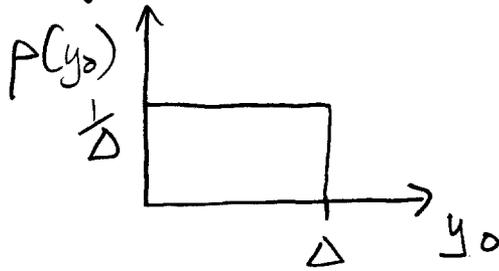
and scan the momentum so that the stopping point is moved by y_0

$$\frac{dT}{dy} = \frac{m}{(y - y_0)^{1/2}}$$

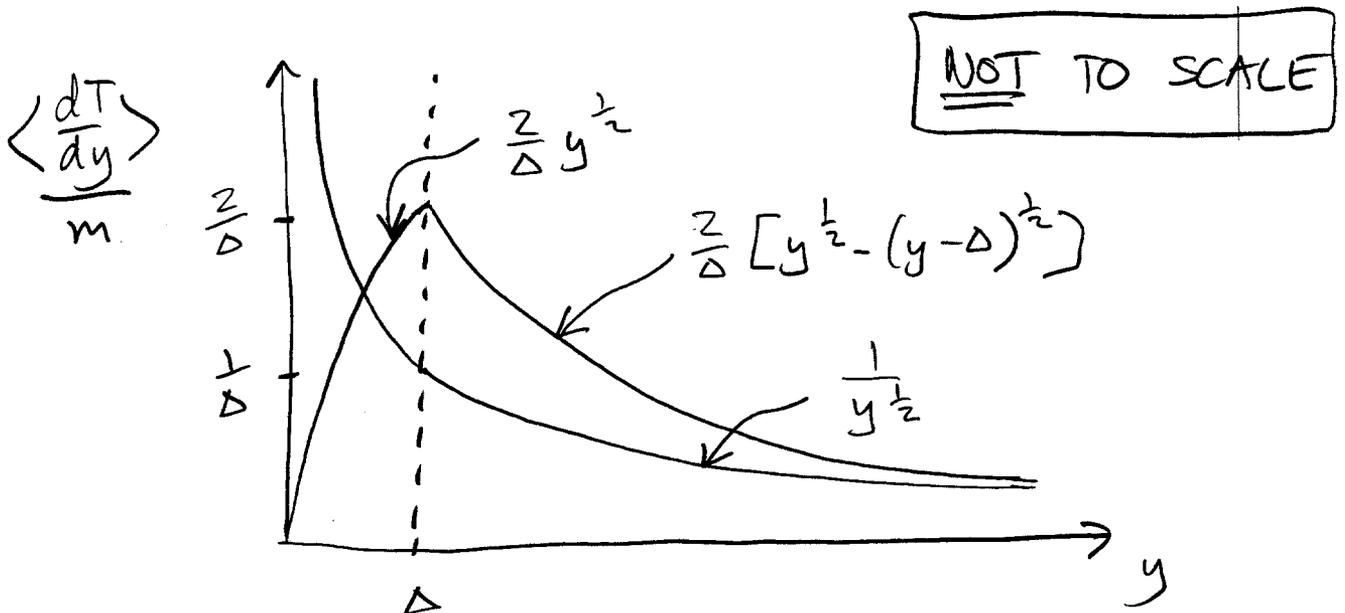
The average rate of energy deposition for a scan represented by $f(y_0)$ is

$$\left\langle \frac{dT}{dy} \right\rangle = \frac{\int \frac{dT}{dy}(y, y_0) f(y_0) dy_0}{\int f(y_0) dy_0}$$

For example, consider a linear scan of depth Δ



$$\begin{aligned} \left\langle \frac{dT}{dy} \right\rangle &= \frac{z_m}{\Delta} y^{\frac{1}{2}} && 0 < y < \Delta \\ &= \frac{z_m}{\Delta} [y^{\frac{1}{2}} - (y - \Delta)^{\frac{1}{2}}] && \Delta < y \end{aligned}$$



Any broadening will have the same general shape

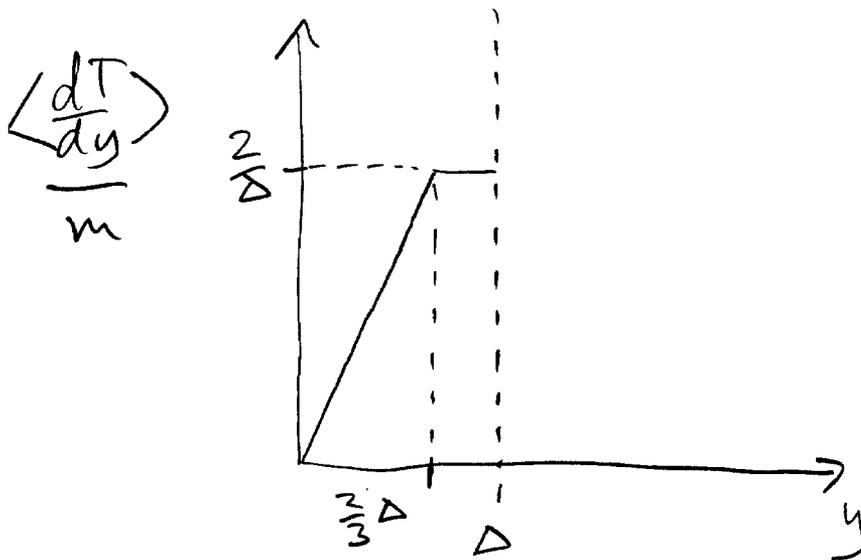
RADIAL SCANNING

Suppose now, for convenience, that $\rho(y_0)$ has been chosen so that

$$\int \rho(y_0) dy_0 = 1$$

$$\rho(y_0) = 0 \quad y_0 < 0, \quad y_0 > \Delta$$

that is, the shape of the curve of has been modified for $0 < y < \Delta$, but the area has been preserved, so that



and assume that the beam has negligible width, so that it may be rotated in a plane, smoothly.

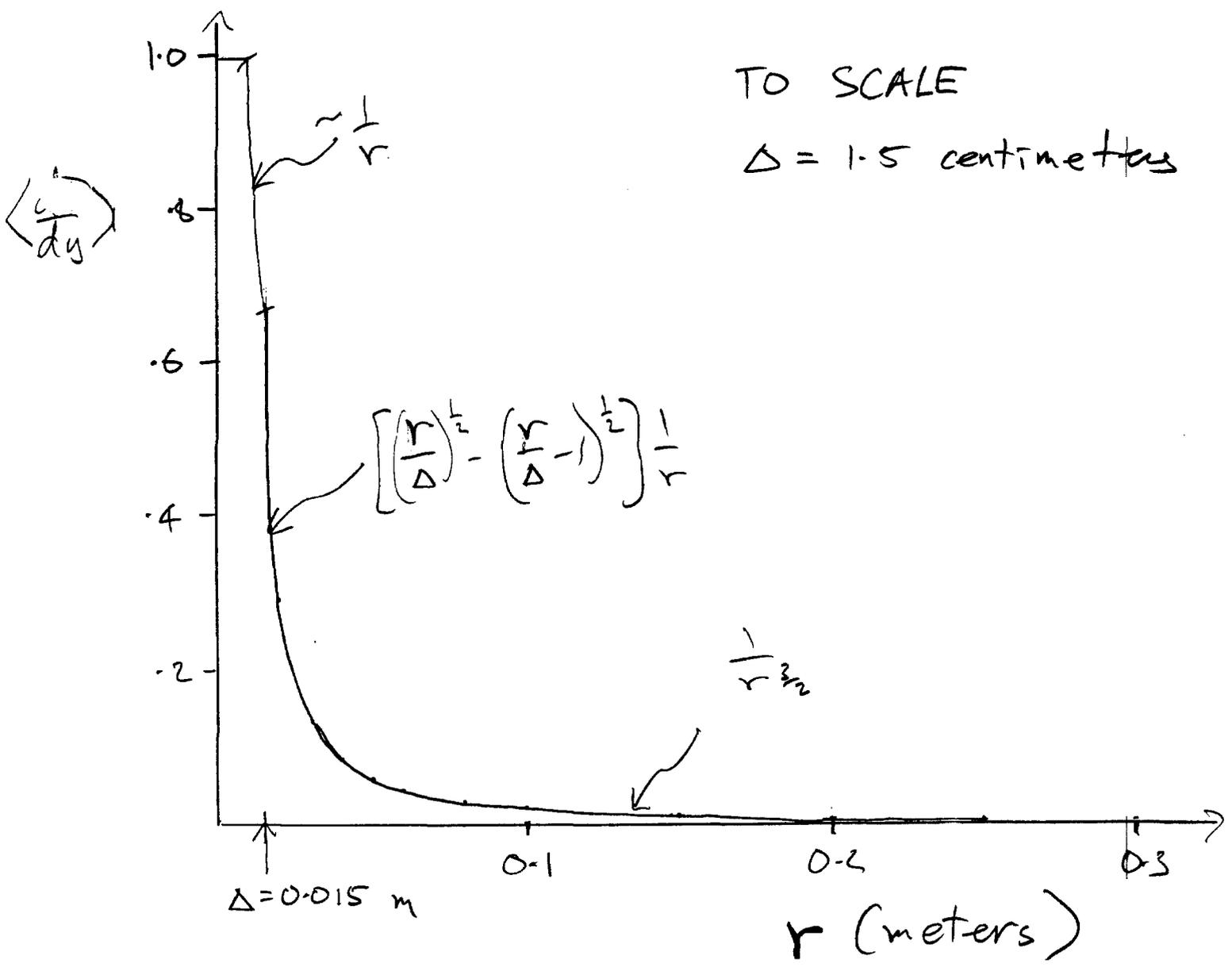
The average rate of energy dissipation, PER UNIT AREA, at radius r , is given by

$$\left\langle \frac{dT}{dy} \right\rangle = \alpha \frac{3}{2} \frac{1}{\Delta^{3/2}} \quad 0 < r < \frac{2}{3} \Delta$$

$$= \alpha \cdot \frac{1}{\Delta^{1/2}} \cdot \frac{1}{r} \quad \frac{2}{3} \Delta < r < \Delta$$

$$= \alpha \cdot \left[\left(\frac{r}{\Delta} \right)^{1/2} - \left(\frac{r}{\Delta} - 1 \right)^{1/2} \right] \frac{1}{r} \quad \Delta < r$$

$$\approx \alpha \cdot \frac{1}{r^{3/2}} \quad \Delta \ll r$$



ACCEPTING A 2 cm APERTURE RADIUS

If PTA runs with a 4 cm (diameter) beam pipe, the following linear components can be used, with some redesign (see below)

- 1) quadrupoles
- 2) beam position monitors
- 3) dipole orbit correctors (can also be easily rewired as quad trims)

Assuming a regular FODO structure throughout, this acceptance constrains L , the half cell length

The maximum β is $\hat{\beta}$, given by

$$\hat{\beta} = \frac{L}{s} \left(\frac{1+s}{1-s} \right)^{\frac{1}{2}}$$

$$s \equiv \sin(\phi_{\frac{1}{2}})$$

half cell phase advance

$\phi_{\frac{1}{2}}$	$\hat{\beta}/L$
30°	3.46
36°	3.34
45°	3.41

Remarkably constant over the range of interest

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As a naive aperture requirement in transverse space only, require $n \sigma$ of beam to be contained

$$a \geq n \sqrt{\frac{\epsilon \hat{\beta}}{(\beta\gamma)_{INS}}} \approx n \sqrt{\frac{3.45 \epsilon}{(\beta\gamma)_I}} L^{\frac{1}{2}}$$

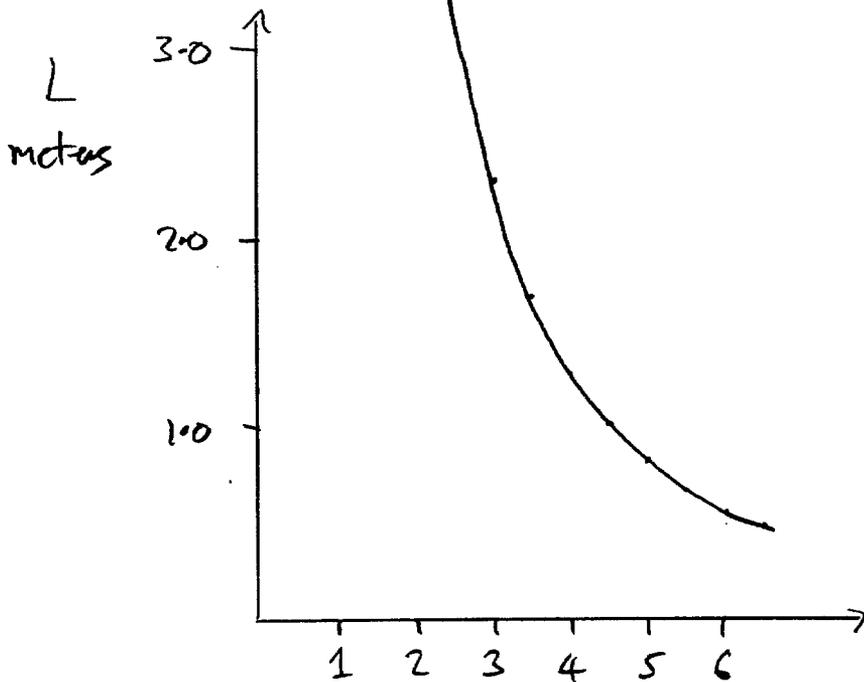
Use $\epsilon = 10^{-6}$ metres (1 σ emittance)

$$(\beta\gamma)_I = 0.179 \quad (T_{INS} = 0.015 \text{ GeV})$$

$$a = 0.02 \text{ metres (Linac upgrade components)}$$

demand

$$L < \frac{20.8}{n^2} \text{ metres}$$



n - number of σ in acceptance

If the lattice is racetrack, with dispersion free straight by having N_A ^{identical} half cells in 360° phase advance arcs,

$$N_A \phi_{\frac{1}{2}} = 360 \text{ degrees}$$

and the maximum dispersion is TWICE that of a matched FODO cell, so

$$\hat{\eta} = 2L\theta \left(\frac{2+S}{2S^2} \right)$$

where θ is the dipole angle

$$\theta = \frac{\pi}{N_A} \text{ radians}$$

N_A	$\phi_{\frac{1}{2}}$	$\hat{\eta}/L$
12	30	2.62
10	36	2.35
8	45	2.13

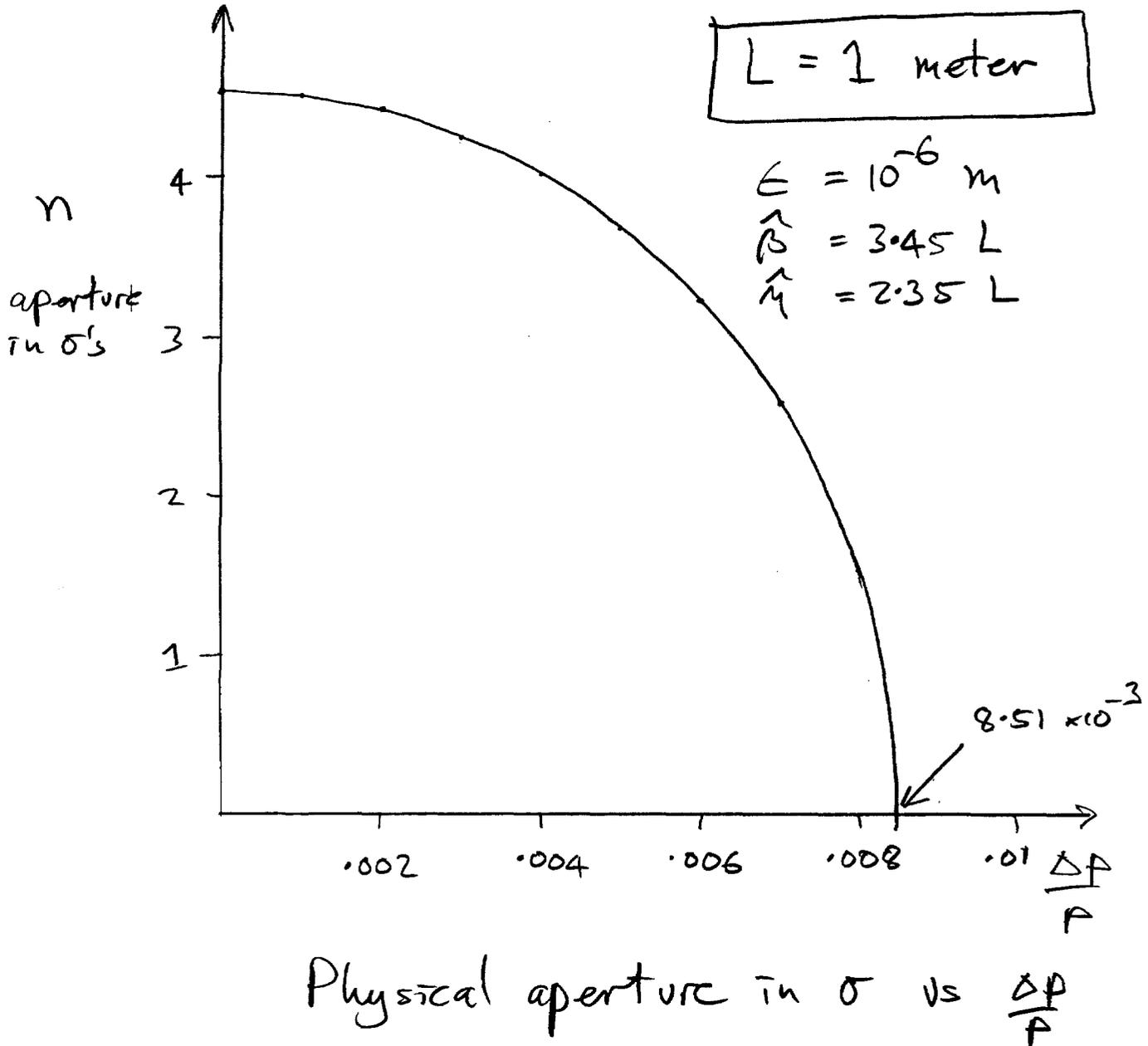
$\pm 10\%$ variation

Taking $L = 1$ meter, and using $\hat{\eta} = 2.35$ m, can draw the physical aperture versus momentum, combining by quadrature

$$0.02^2 = \left[\hat{\eta} \left(\frac{\Delta p}{p} \right) \right]^2 + [n\sigma]^2$$

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$$\hat{\sigma} = \left(\frac{3.45 \times 10^{-6}}{0.179} \right)^{\frac{1}{2}} = 4.39 \times 10^{-3} \text{ m}$$



Conclude that $L=1$ is about right - good enough for a first pass.

SELECTING N_A , NUMBER OF ARC HALF CELLS

The aperture discussion above is only weakly dependent on N_A , which is mainly determined by half cell packing

Assume that $B_{\text{DIPOLE}} = 1.215$ Tesla at $T = 0.25$ GeV

then

$$\rho = 2.0 \text{ meter}$$

and

$$L_D = \theta \rho = \frac{2\pi}{N_A}$$

[1 dipole per half cell]

Assuming that $B'_{\text{QUAD}} = 21.6$ T/m @ $T = 0.25$ GeV,
from normal operating specs of linac upgrade quad,
and

$$B' L_Q f = \frac{P}{.299792}$$

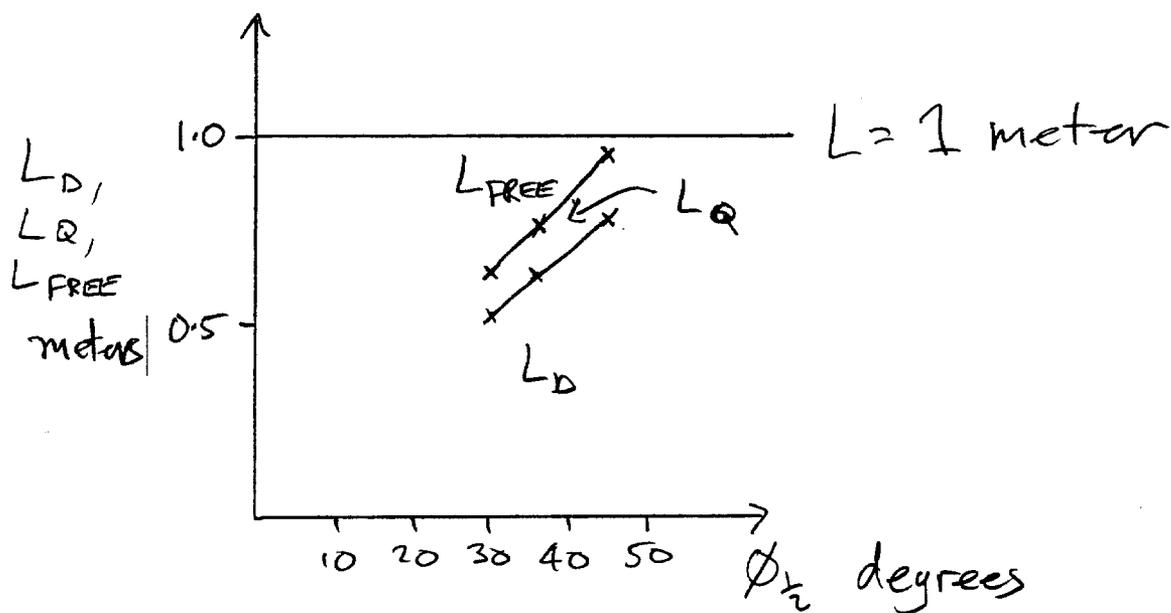
and

$$\frac{1}{f} = \frac{2 \sin(\phi_{\frac{\pi}{2}})}{L}$$

then

$$L_Q = \frac{P}{.299} \frac{1}{B'} \frac{2 \sin(\phi_{\frac{\pi}{2}})}{L} = 0.225 \sin\left(\frac{2\pi}{N_A}\right)$$

N_A	$\phi_{\frac{1}{2}}^\circ$	$L_D(m)$	$L_Q(m)$	$L_{FREE}(m)$
12	30	.524	.113	.363
10	36	.628	.132	.240
8	45	.785	.159	.056



* This suggests that $N_A = 10$, $\phi_{\frac{1}{2}} = 36^\circ$ is the correct choice for $L = 1$

* If the half cell length must decrease for better aperture, go to $N_A = 12$, $\phi_{\frac{1}{2}} = 30^\circ$

* Remember that $L_Q \sim \frac{1}{L}$

* Of course, N_A could be an odd number

SELECTING N_s , NUMBER OF STRAIGHT HALF CELLS

* Need a long enough straight, and enough phase advance, for ...

- injection
- extraction
- RF

* Must not let the straight phase be a multiple of 90° (or total tune will be on a half integer)

$$N_s \phi_{\frac{1}{2}} \neq \text{integer} * 90^\circ$$

N_A	$\phi_{\frac{1}{2}}$	N_s (OK)
12	30	1 2 4 5 7 8 10
10	36	1 2 3 4 6 7 8 9
8	45	1 3 5 7 9

* Unless $N_A = 8$, the best solution appears to be $N_s = 4$ since then have



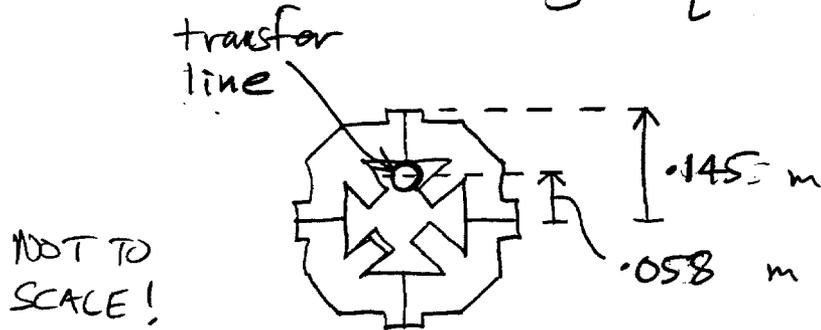
K - kicker
S - septum

$$\phi_{\text{SEPTUM}} - \phi_{\text{KICKER}} = 2\phi_{\frac{1}{2}}$$

$$\sin(\Delta\phi) = 0.866 \quad \text{or} \quad 0.951$$

INJECTION / EXTRACTION

With the Linac Upgrade (LU) quads, can probably inject/extract through quad poles



to do this, need a septum angle

$$* \theta_{\text{SEPTUM}} = \frac{0.058}{\frac{1}{2} L} \approx 0.116 \text{ milliradians in } \lesssim 1 \text{ metre}$$

$$* A_{\text{SEPTUM}} \lesssim \frac{L}{0.116} \approx 8.6 \text{ meters}$$

$$* \text{ This requires a field of } \frac{2.43}{8.6} \approx 0.28 \text{ Tesla}$$

* If the beam needs to be offset by 0.02 m in the septum, need a kicker angle

$$\theta_{\text{KICKER}} \approx \frac{0.02}{\beta_{\text{typ}}} \approx \frac{0.02}{2} \approx 10 \text{ milliradians}$$

which means about 0.042 Tesla (@ extraction)

DYNAMIC APERTURE

dominated by sextupoles with high quality dipoles, etc

$$\Delta x' = -s x^2, \quad s \sim \frac{1}{\eta \beta}$$

In normalised phase space, $x_N = \frac{x}{\sqrt{\beta}}$, $x_N' = \sqrt{\beta} x'$

$$\Delta x_N' = -S_N x_N^2$$

$$S_N = \beta^{3/2} s, \quad S_N \sim \frac{\beta^{1/2}}{\eta}$$

it is easy to show that normalised dynamic aperture scales inversely with S_N

$$a_{ND} \sim \frac{1}{S_N}$$

so, unnormalised,

$$a_D \sim \frac{\beta^{1/2}}{S_N} \sim \eta$$

Insofar as η is not especially small, dynamic aperture due to sextupoles should not be a problem

Although

- *) Eddy currents are strong
- *) sextupole layout is relatively irregular

LINAC UPGRADE COMPONENTS

Quad

$$L_{\text{IRON}} = 0.07 \text{ m}$$

$$L_{\text{MAG}} = 0.0853 \text{ m}$$

3% duty factor @ 15 Hz

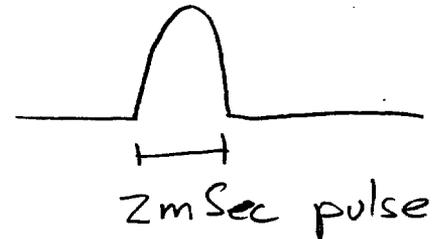
$$B' = 21.6 \text{ T/m}$$

$$P = IV = 1.58 \text{ kW}$$

$$\text{Eddy loss} \quad \begin{array}{l} -265 \text{ kW (Fe)} \\ -0.35 \text{ kW (Cu)} \end{array}$$

$$\langle P \rangle = 0.040 \text{ kW}$$

Iron is water-cooled, coil is not



FOR PTA

- * lengthen magnet
- * redesign coil with water-cooling
- * Eddy losses are about the same @ 60 Hz, 100% duty factor
- * should examine decapole components in tracking
 - probably OK
- * are eddy currents important, eg in dipole/quad tracking?
- * could begin redesign work immediately!

BPMs

- * will work fine in PTA, as is
- * fit inside quad poles - take "no" space

Dipole correctors (window frame)

$$L_{DC} \approx 0.09 \text{ m}$$

are run DC with no cooling, except for vanes

$I = 6A$ "too hot to touch", saturation begins

$$I = 10A \quad B \approx 0.1 \text{ Tesla}$$

$R_{coil} \sim 1\Omega$ with 16 gauge wire

$$\theta_{DC} \approx 2.0 \text{ mrad @ } 5Amps, T = 0.250 \text{ GeV}$$

compare with quad misalignment of 1 mm

$$\theta_Q = \frac{10^{-3}}{f} = \frac{2 \sin(36) \times 10^{-3}}{L} = 1.18 \text{ mrad}$$

LU correctors are a good match for PTA

For PTA

- * modify wire gauge, iron thickness ?
- * use also for quad trim?

$$\frac{1}{f} \approx \frac{B' L_0}{P/3} = \frac{0.1}{0.05} \cdot 0.1 \cdot \frac{.3}{.729} \approx 0.08 \text{ m}^{-1}$$

good for a few % tune changes (if everything)

BOSES AND CORRECTION CIRCUITS

- * Put quads and dipoles on same resonant bus, but worry about tracking errors through
 - 1) relative saturation
 - 2) eddy currents
- * If necessary tune corrections are large
 - 1) take 2 pairs of F and D quads off main bus
 - OR
 - 2) modify number of turns on quads
- * If tune corrections are small, use "dipole correctors" (with reconnected windings)
- * In general, all correctors (quad, dipole correctors) have a strength that is a function of B

$$S_{\text{CORR}} = S_c(B) = S_{c0} + S_{c1} B + S_{c2} B^2 + \dots$$

where

$$B = \frac{1}{2}(B_E + B_I) - \frac{1}{2}(B_E - B_I) \cos(2\pi f_R t)$$

and injection field B_I is fixed, with extraction field B_E slowly varying.

then in general have

$$S_c = \sum_{n=0}^{n_{\text{max}}} S_{cn}(B_E) \cos(2\pi n f_R t)$$

Having found S_{e0}, \dots, S_{en} etc, by machine observation, can derive their Fourier counterparts \tilde{S}_{en} , allowing correction during energy scanning in an easy way

- * It may be necessary to include a phase offset as well, e.g. if \dot{B} is important
- * chromaticity sextupoles can probably be powered in series, on a single resonant bus - out of phase from the main bus, since

$$I_{SEXT} = \alpha_0 B + \alpha_1 \dot{B} = \tilde{\alpha}_0 + \tilde{\alpha}_1 \cos(2\pi f_R t + \phi)$$